Course Remote Sensing and Climate

Summer 2026
University of Frankfurt

2. Day (R. Hollmann)

- ➔ Content and Structure of Course
- ➔ Motivation and background
- ➔ A short introduction in Physics
- ➔ **Radiative Transport and retrieval basics**
- ➔ Satellite orbits and instruments
- ➔ Climatologies based on satellite instrument and usage

Notes to PDF-Version

This PDF is based on the presentation given in the summer term 2025 of University Frankfurt as part of the course „remote sensing and climate“.

It is meant to recall the content the content of the lecture and is for the personal use of the participants of the course and <u>not</u> for a wider audience.

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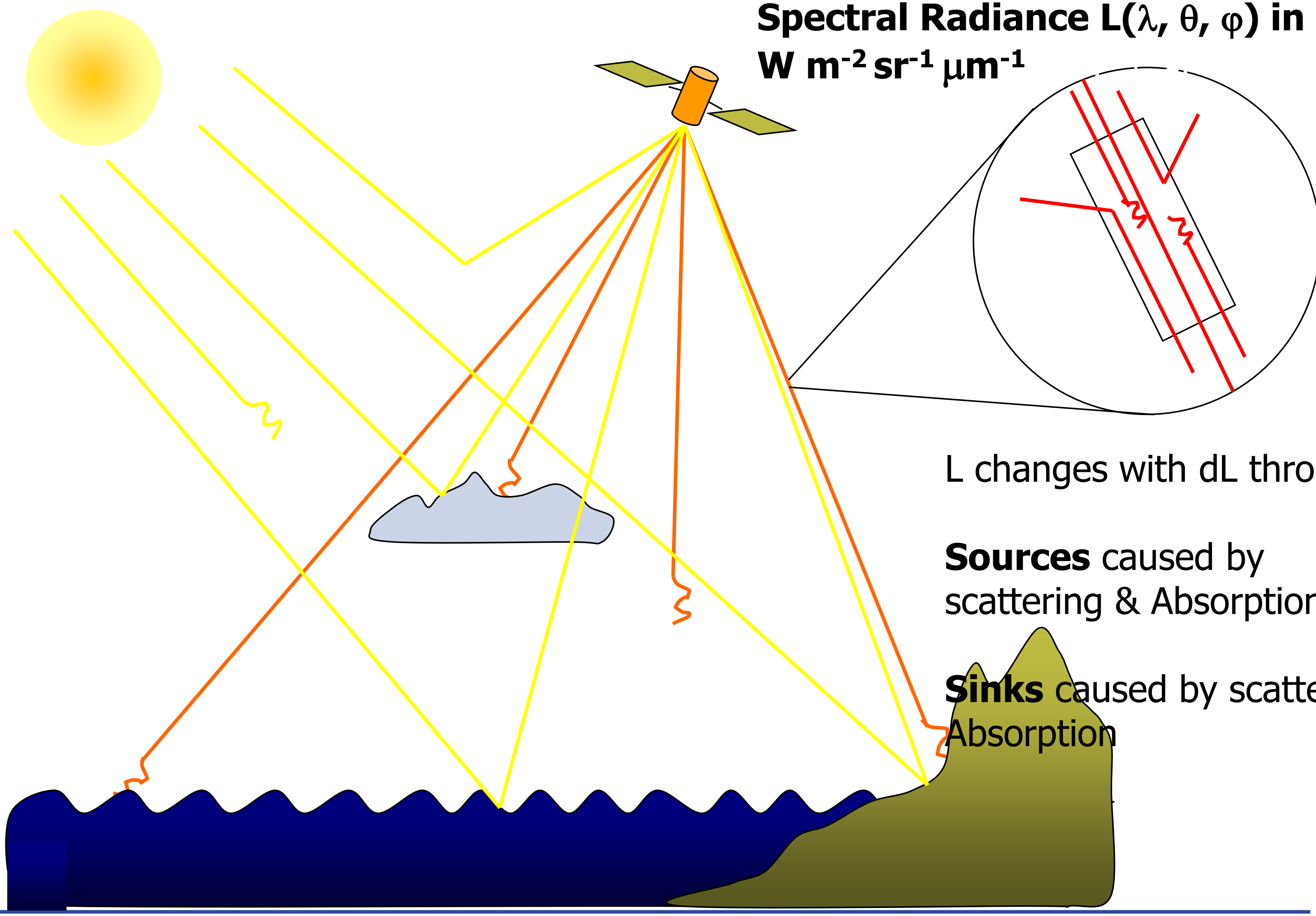
Radiative Transport and retrieval basics

Content

- ➔ Radiative Transfer Theory
- ➔ 3 special solution of the Radiative Transfer Equation
- ➔ Solution of the Radiative Transfer Equation for Microwaves

Radiative Transfer (I)

Spectral Radiance L(λ, θ, φ) in W m^{-2} sr^{-1} μm^{-1}

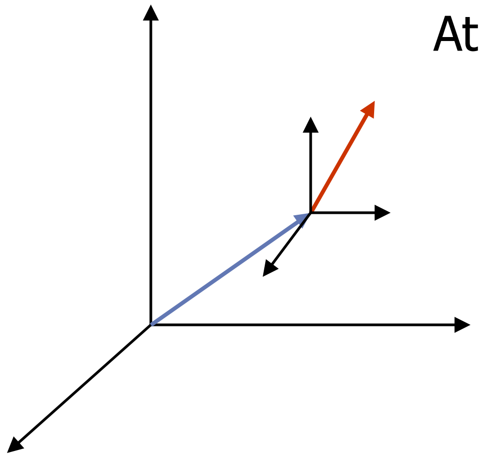


L changes with dL through:

Sources caused by scattering & Absorption

Sinks caused by scattering & Absorption

Radiative Transfer (II)



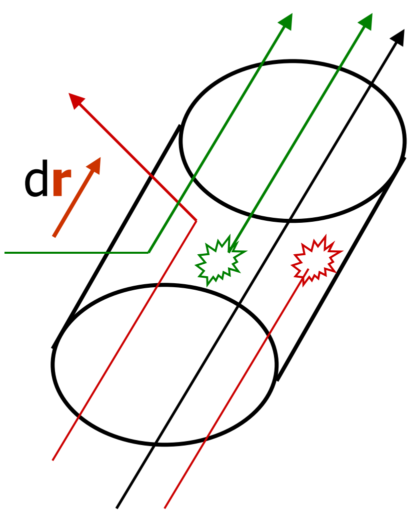
At any location X(x,y,z) along the vector $\boldsymbol{r}$ (r,θ,φ):

$$dL(\lambda,X,r) = -\sigma_e(\lambda,X)L(\lambda,X,r)dr + J(\lambda,X,r)dr$$

1. Term presents a loss of photons

$$\sigma_e(\lambda,X) = \sigma_a(\lambda,X) + \sigma_s(\lambda,X)$$

σ_e = Extinction coeffient
σ_a = Volume absorption coefficient
σ_s = Volume scattering coefficient
Unit: 1/Length

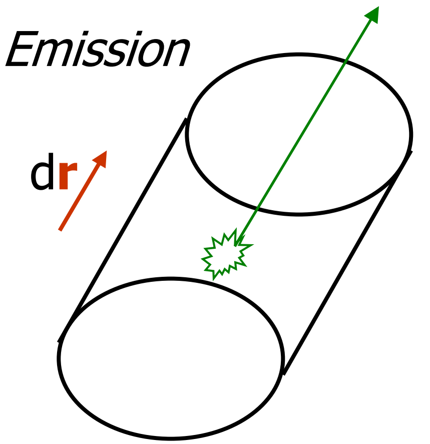


2. Term presents a source of photons

$$J = J_{th} + J_{scat}$$

Emission Scattering

Radiative Transfer (III)

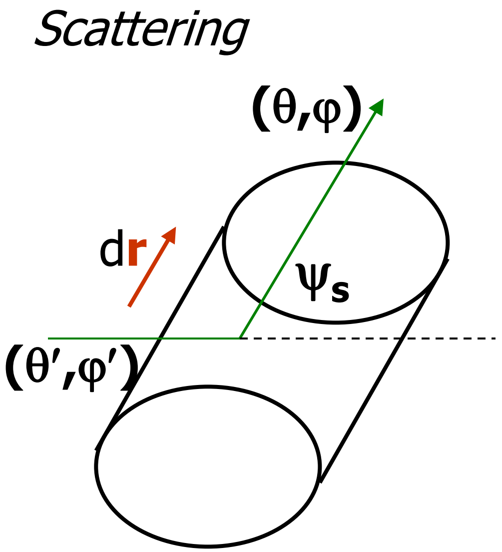


$$J_{th} = \sigma_a(\lambda, X)B(\lambda, T(X))$$

Emission along the path r is given by the Planckfunction for T(X)

T(X) = Temperature at location X

$\sigma_a(\lambda, X) = \varepsilon(\lambda, X)$ Thermal Emission (Kirchhoff)



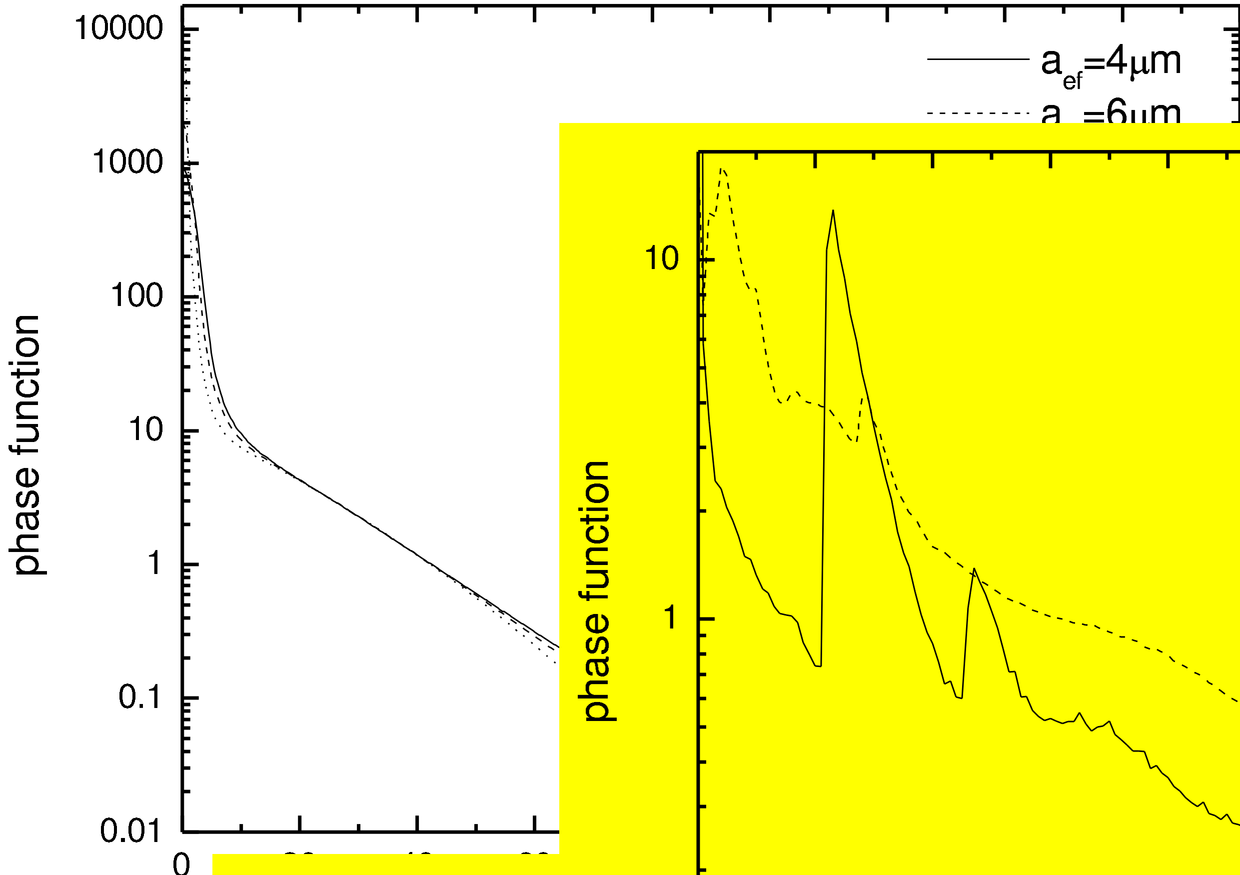
$$J_{scat}(\lambda, X) = \sigma_s(\lambda, X)\frac{1}{4\pi}\int_{4\pi} P(r', r, \lambda, X)L(r', \lambda, X)d\Omega'$$

P = Phasefunction which is the probability that a photon will be scattered from r′ in direction of r.
Definition of scattering angle from direction **(θ′,φ′)** in direction (θ**,**φ):

It is: $\cos\Psi_s = \cos\theta\cos\theta' + \sin\theta\sin\theta'\cos(\varphi - \varphi')$

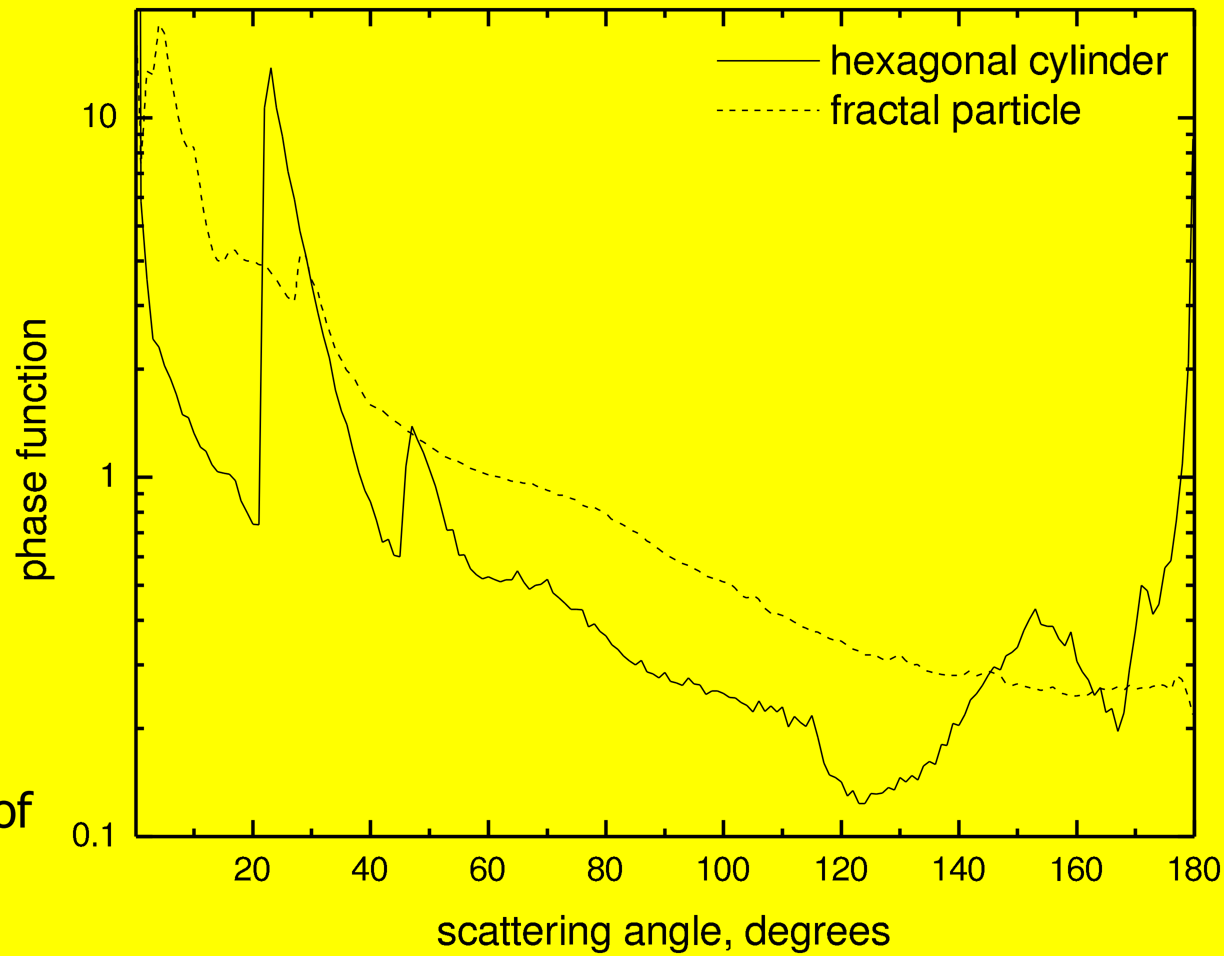
And also: $P(r', r, \lambda, X) = P(\theta', \theta, \varphi', \varphi, \lambda, X) = P(\Psi_s, \lambda, X)$

Phasefunction (I)



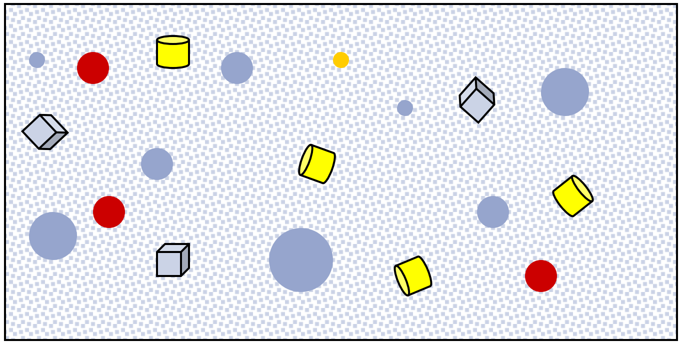
Liquid phase, different effective radii of droplets

Solid phase, different types of ice crystalls



Optical thickness & Single Scattering Albedo

Optical properties of particles are defined



- Are independent from radiation

$$\sigma_e, \sigma_a, \sigma_s, \varepsilon, P(\Psi_S)$$

Optical thickness of atmospheric layer :

$$\delta(\lambda, z) = \int_z^{z_\infty} \sigma(\lambda, z')dz'$$

Probability that a scattering or absorption event happens if a photon meets a particle (**single scattering albedo**):

$$\omega_0 = \frac{\sigma_s(\lambda, X)}{\sigma_e(\lambda, X)}$$

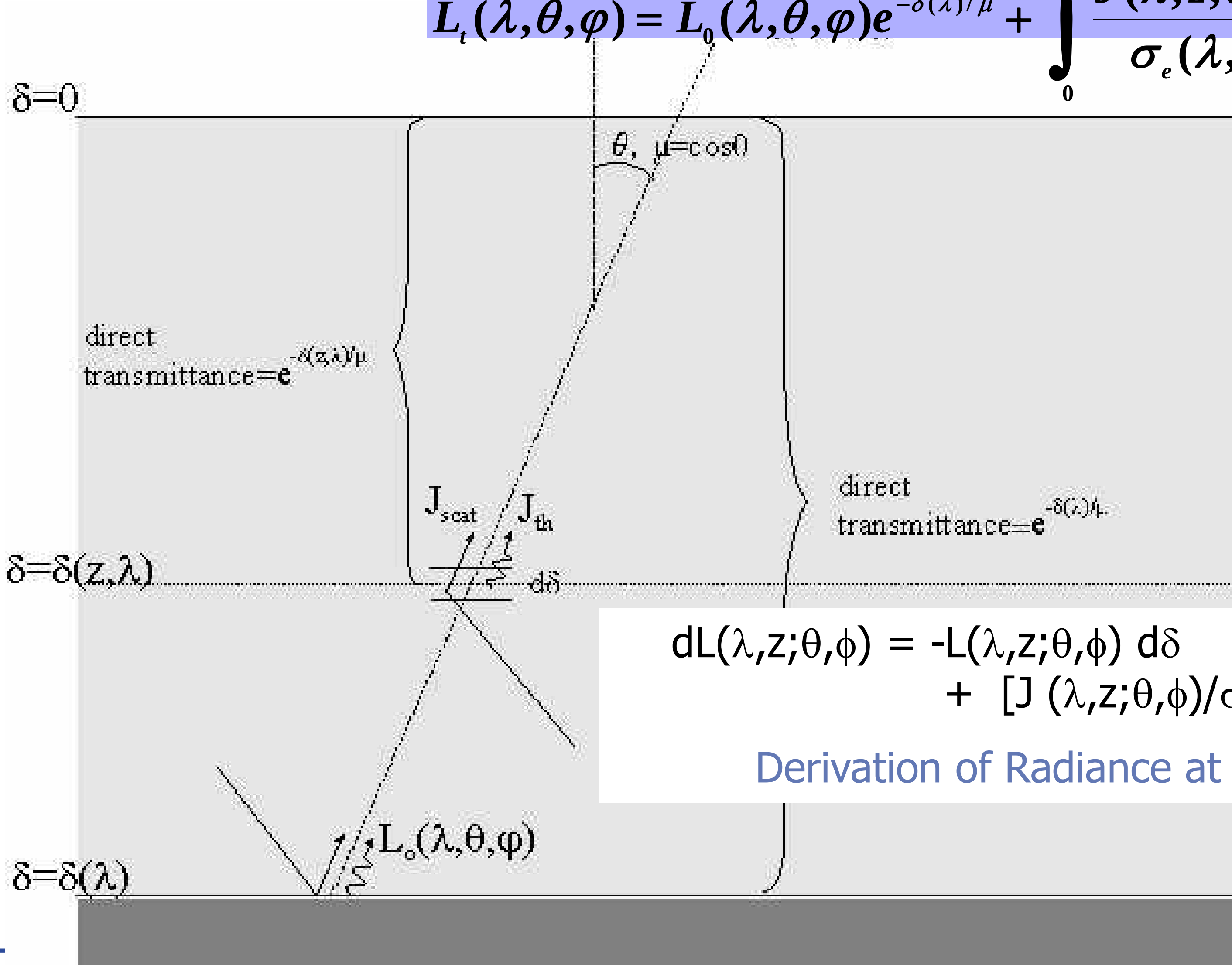
wenn $\omega_0 = 1$ ...

wenn $\omega_0 = 0$ ...

Radiance at the Top of the atmosphere (I)

Radiance at the Top of the atmosphere

$$L_t(\lambda,\theta,\varphi) = L_0(\lambda,\theta,\varphi)e^{-\delta(\lambda)/\mu} + \int_0^{\delta(\lambda)} \frac{J(\lambda,z;\theta,\varphi)}{\sigma_e(\lambda,z)} e^{-\delta(\lambda,z)/\mu} \frac{d\delta}{\mu}$$



$$dL(\lambda,z;\theta,\phi) = -L(\lambda,z;\theta,\phi)\,d\delta + [J(\lambda,z;\theta,\phi)/\sigma_e(\lambda,z)]\,d\delta$$

Derivation of Radiance at height z

Simplified Solutions of RTE

Now we look into simplified solutions of the Radiative Transfer Equation (RTE) at the top of the atmosphere.

These solutions are characteristic for main application areas (in retrievals) in satellite remote sensing.

We introduce now a few constraint to simplify the RTE that the principle of a dedicated retrieval becomes visible.

Thus, for IR and Visible wavelengths we define:

Case #1 – No Emission and No Scattering along the path

Case #2 – Only Emission as source term

Case #3 – Only single scattering as source term

A simplified solution for Microwave will come afterwards.

Simplified Solution of RTE (Case #1)

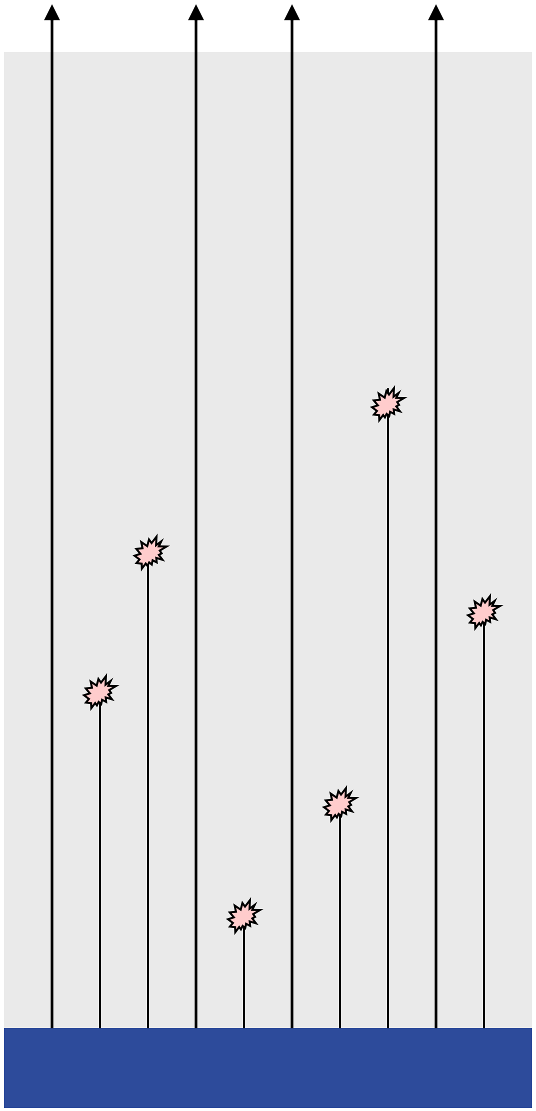
Case #1

No Emission or Scattering along the path, i.e.:

We have a wavelength where B(λ,T(z)) ~ 0
and there are no scattering particles.

Radiance at the Top of the Atmosphere

$$L_t(\lambda,\theta,\varphi) = L_0(\lambda,\theta,\varphi)e^{-\delta(\lambda)/\mu} + \int_0^{\delta(\lambda)} \cancel{\frac{J(\lambda,z;\theta,\varphi)}{\sigma_e(\lambda,z)}} e^{-\delta(\lambda,z)/\mu}\frac{d\delta}{\mu}$$



δ ~ ?

Simplified Solution of RTE (Case #1)

Case #1

No Emission or Scattering along the path, i.e.:

We have a wavelength where $B(\lambda,T(z)) \sim 0$ and there are no scattering particles.

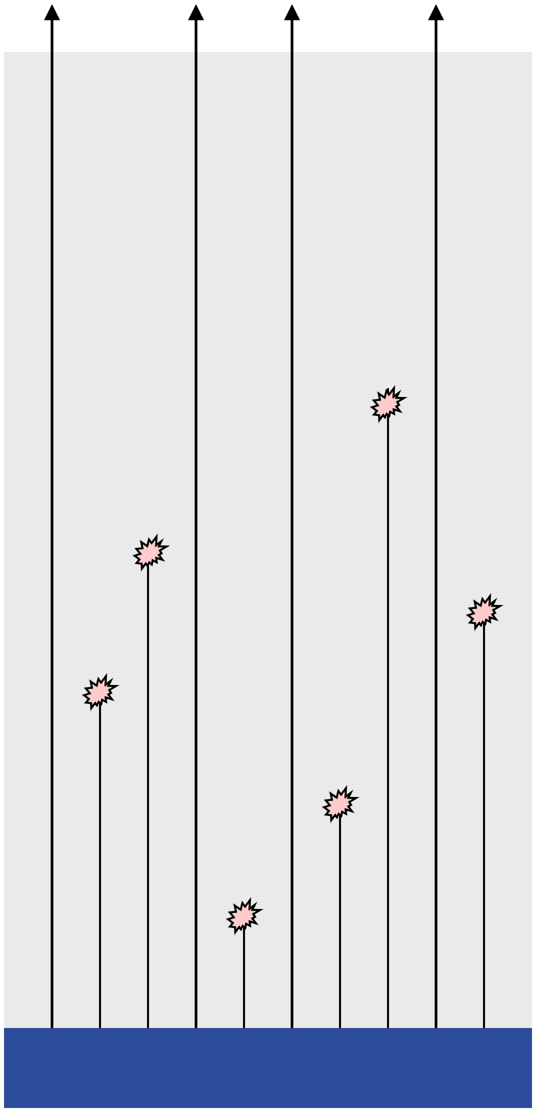
Solution:

$$L_t(\lambda,\theta,\varphi) = L_0(\lambda,\theta,\varphi)e^{-\delta(\lambda)/\mu}$$

This is known as law from Bouguer-Lambert-Beersches

if $\theta = 0$

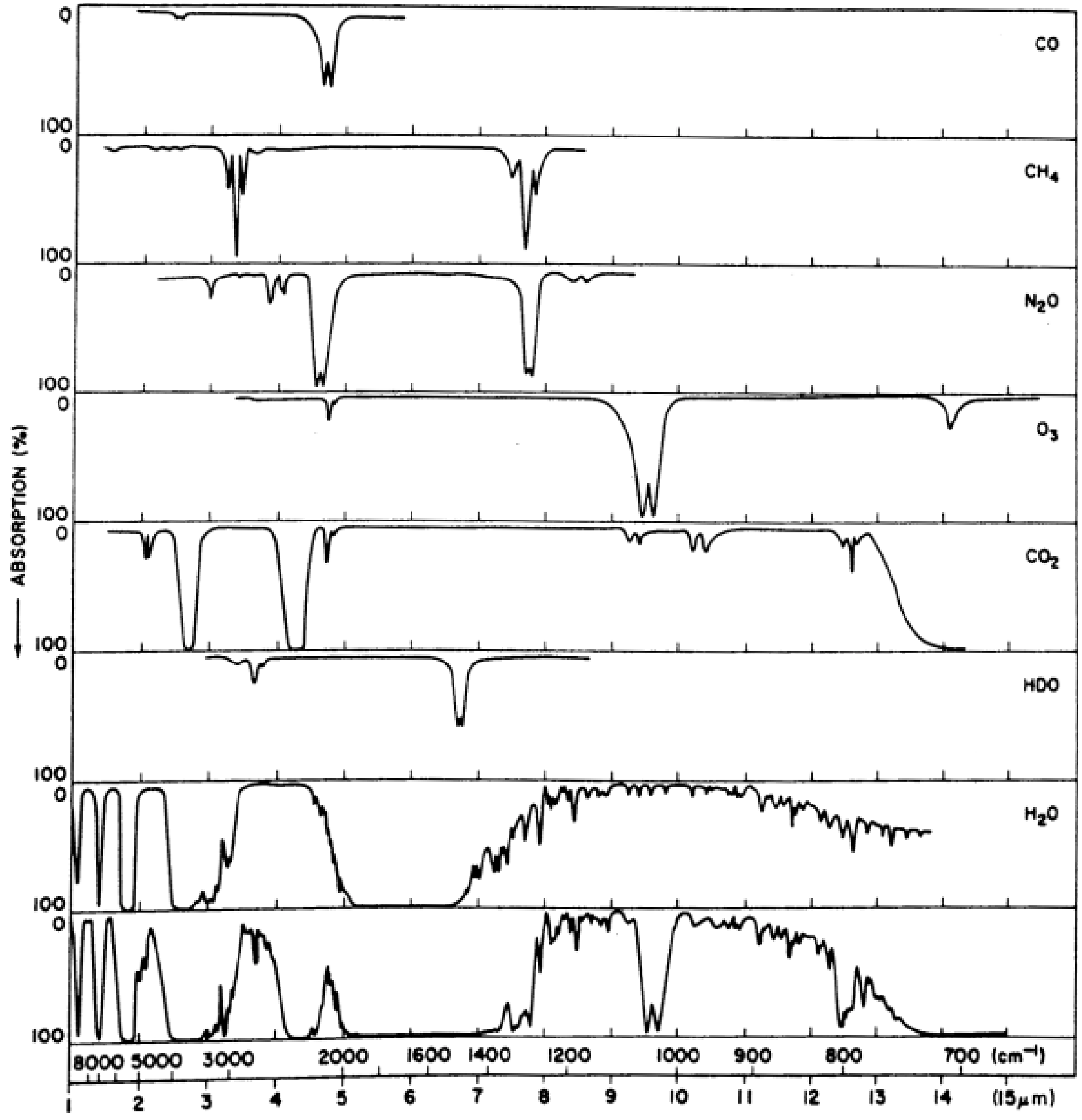
$\delta =$	0.01	0.1	1.0	7
$e^{-\delta} =$	99%	90,5%	36,8%	0,1%



$\delta \sim ?$

Application of Case #1

For suited wavelengths it is possible to direcltly relate the content or concentration of gase to the measured radiance

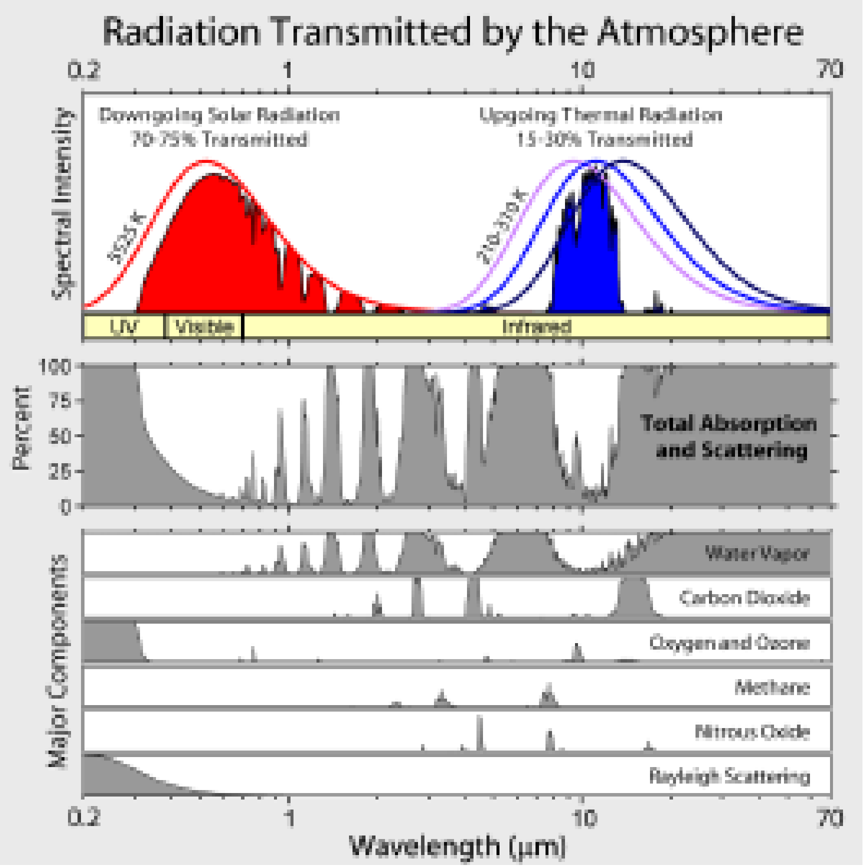


Important gas absorption lines

Die verschiedenen Luftmoleküle absorbieren in spezifischen Banden. Die wichtigsten im Infraroten Spektralbereich sind:

- Kohlendioxid (CO_2): 4,3 µm und 15 µm
- Wasserdampf (H_2O): 6,3 µm, > 10 µm
- Sauerstoff (O_2): UV
- Ozon (O_3): UV, 9,6 µm
- Methan (CH_4): 7,7 µm
- Distickstoffmonoxid (N_2O): 7,8 µm

Um Informationen über die Verteilung und Konzentration dieser Gase zu erhalten, verwendet Satelliteninstrumente Kanäle in genau diesen Absorptionsbanden.



Transmissionsspektrum der Atmosphäre. Rot: Eingestrahltes Sonnenlicht. Blau: Emittierte Wärmestrahlung der Erde. Graue Kurven: Absorptionsspektren der relevanten Gase in der Atmosphäre.

Simplified Solutions of RTE

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Simplified Solution of RTE (Case #2)

Case #2

Only Emission along the path, i.e. emission is the only source for photon, there is no scattering, thus…

$$J(\lambda, z) = \sigma_a(\lambda, z) B(\lambda, T(z))$$

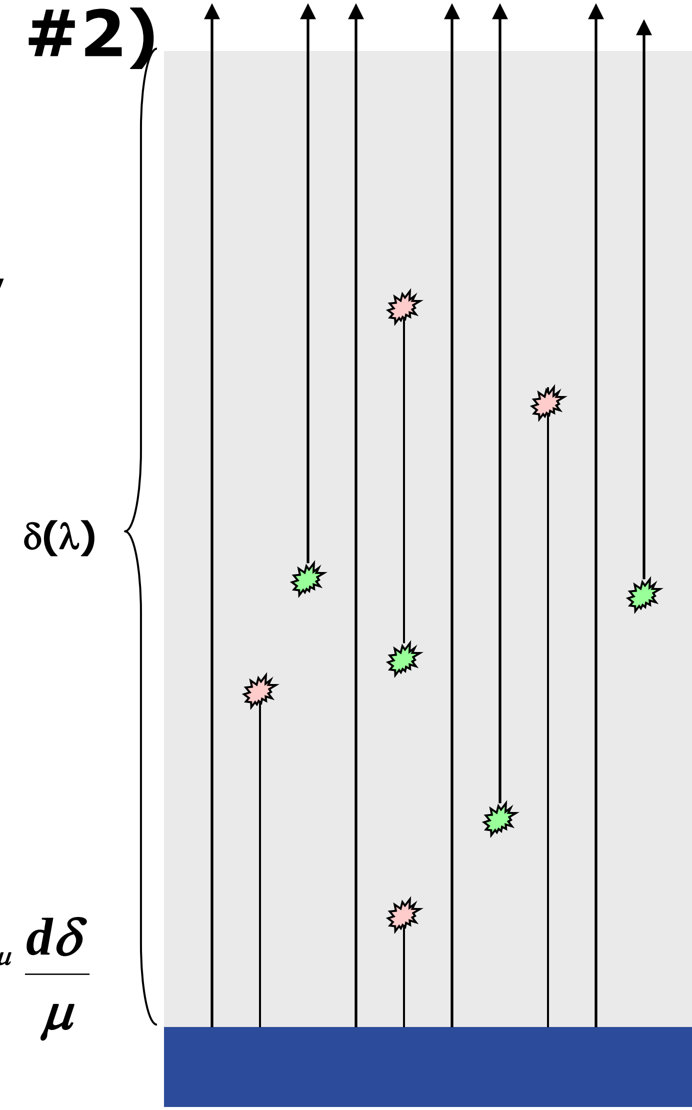
and $\quad \sigma_e(\lambda, z) = \sigma_a(\lambda, z)$

and $\quad L_0(\lambda, \theta, \varphi) = \varepsilon_S(\lambda, \theta) B(\lambda, T_S)$

This solution is called Schwarzschild Equation

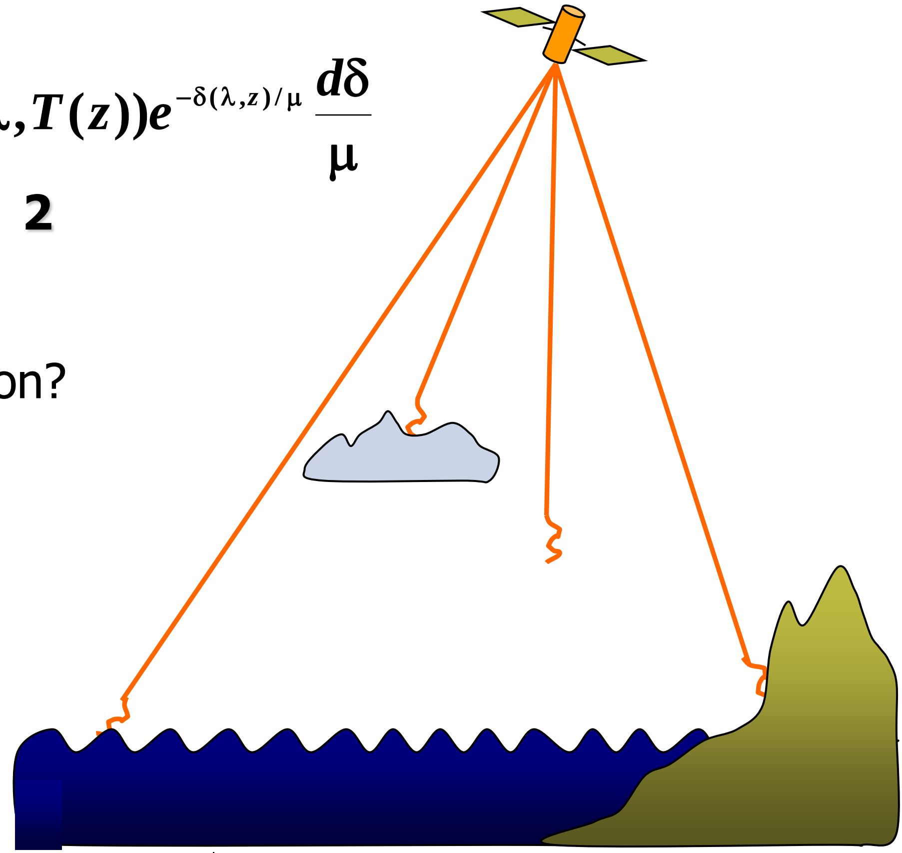
$$L_t(\lambda,\theta,\varphi) = L_0(\lambda,\theta,\varphi) e^{-\delta(\lambda)/\mu} + \int_0^{\delta(\lambda)} \frac{J(\lambda,z;\theta,\varphi)}{\sigma_e(\lambda,z)} e^{-\delta(\lambda,z)/\mu} \frac{d\delta}{\mu}$$

$$L_t(\lambda,\theta,\varphi) = \varepsilon_S(\lambda,\theta) B(\lambda,T_S) e^{-\delta(\lambda)/\mu} + \int_0^{\delta(\lambda)} B(\lambda,T(z)) e^{-\delta(\lambda,z)/\mu} \frac{d\delta}{\mu}$$



Simplified Solution of RTE (Case #2)

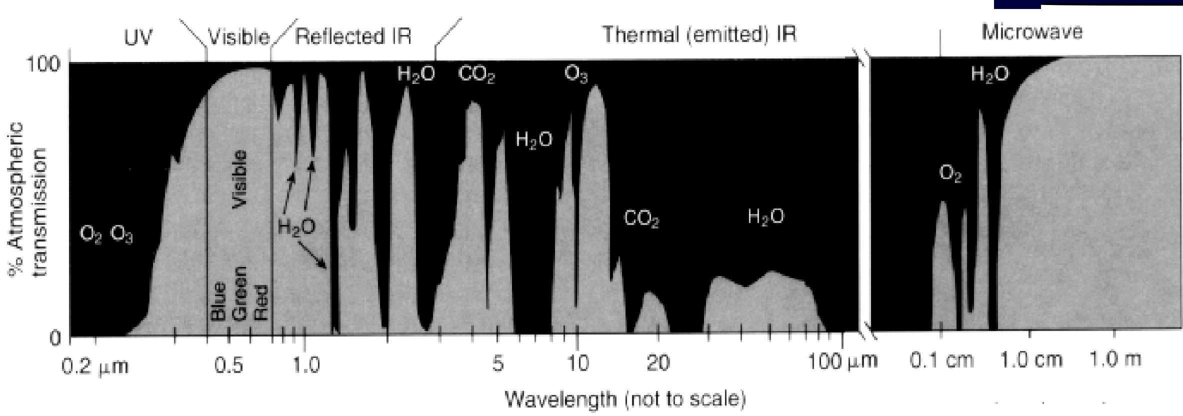
$$L_t(\lambda,\theta,\varphi) = \underbrace{\varepsilon_S(\lambda,\theta)B(\lambda,T_S)e^{-\delta(\lambda)/\mu}}_{1} + \underbrace{\int_0^{\delta(\lambda)} B(\lambda,T(z))e^{-\delta(\lambda,z)/\mu}\frac{d\delta}{\mu}}_{2}$$



For which Wavelength can we use this solution?

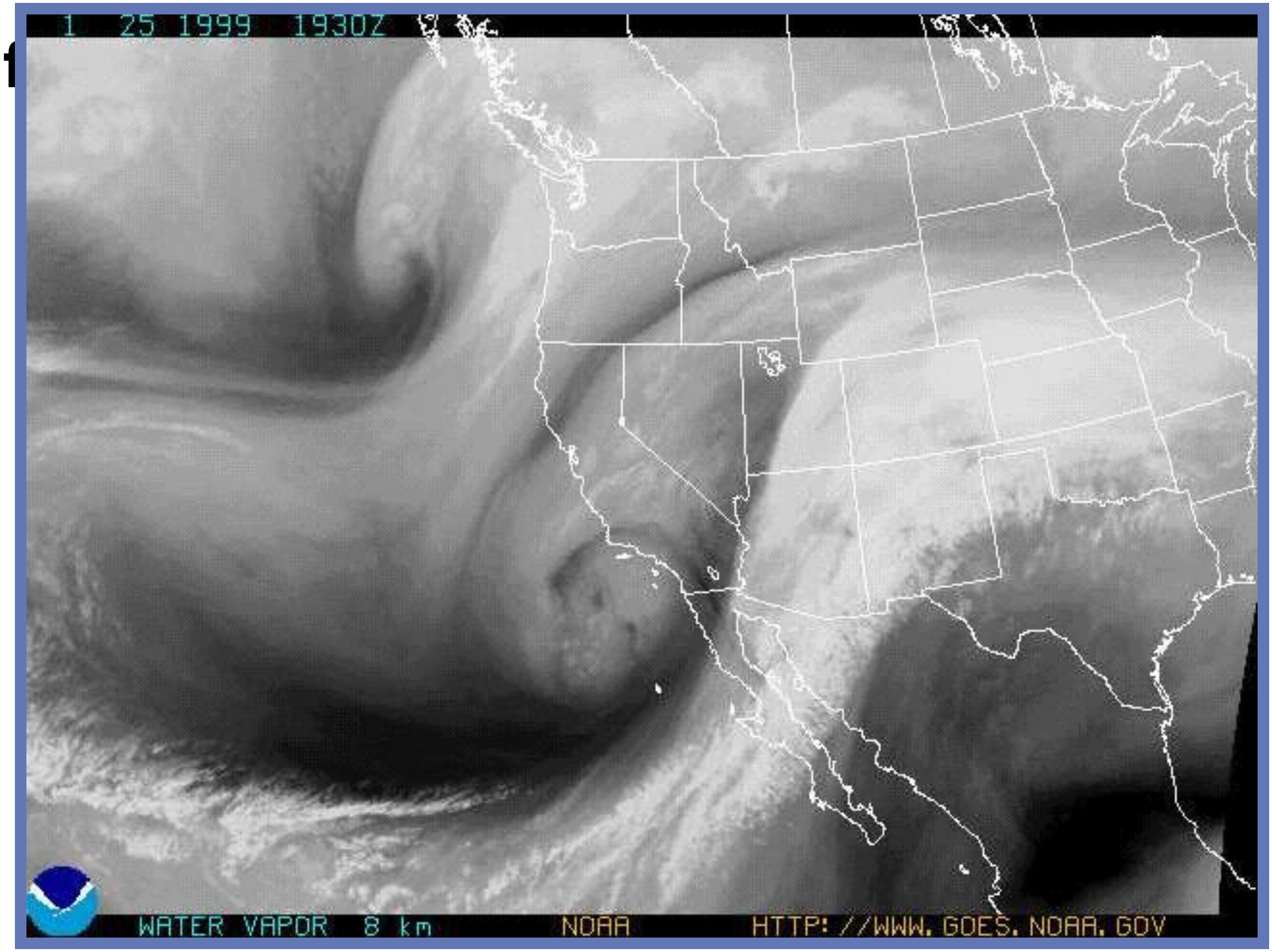
Where dominates Term 1?

Where dominates Term 2?

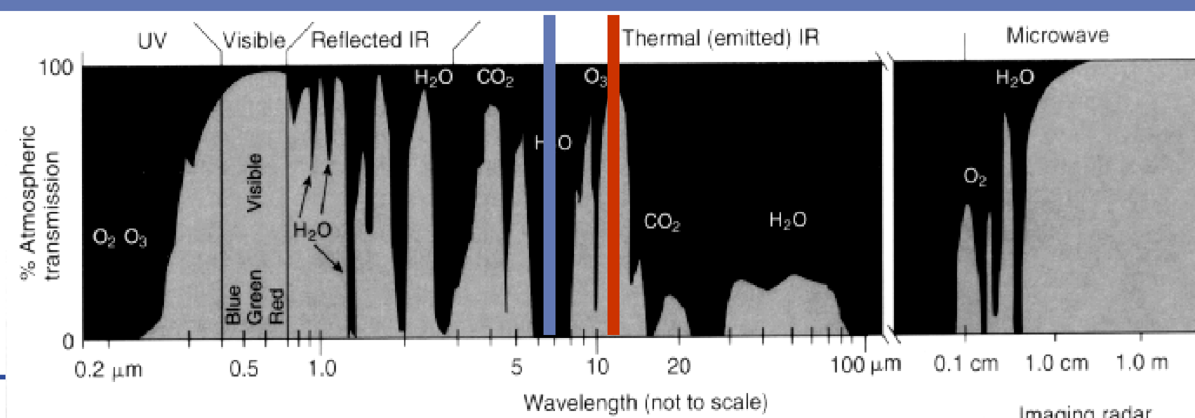


Maximum of Planckfunction with ~300K at 9,66 µm.

Simplif

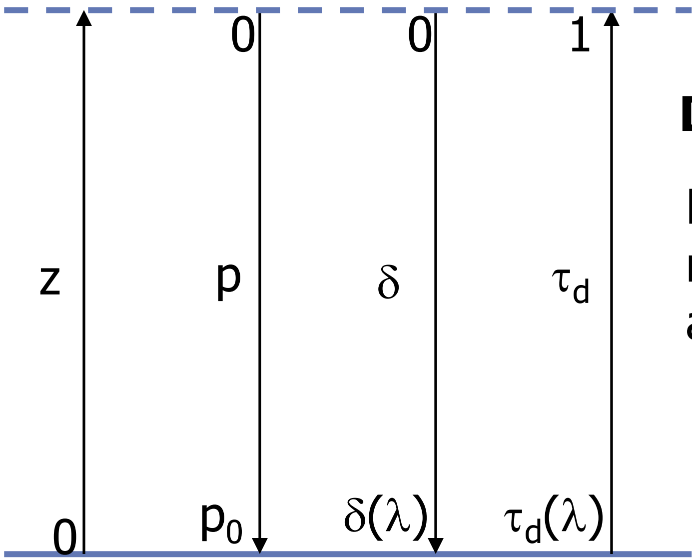


1 25 1999 1930Z
WATER VAPOR 8 km
NOAA
HTTP://WWW.GOES.NOAA.GOV



UV
Visible
Reflected IR
Thermal (emitted) IR
Microwave
100
0
% Atmospheric transmission
O_2 O_3
Visible
Blue
Green
Red
H_2O
H_2O
CO_2
O_3
CO_2
H_2O
H_2O
O_2
0.2 µm
0.5
1.0
5
10
20
100 µm
0.1 cm
1.0 cm
1.0 m
Wavelength (not to scale)
Imaging radar

Schwartzschild's Equation can be simplified with a new vertical variable…



Direct Transmission, $\tau_d = e^{-\delta(\lambda,p)/\mu}$

Probability that a photon at wavelength λ moves directly (without interaction) from a level p to the top of the atmosphere.

$$\frac{d\tau_d(\lambda,p)}{dp} = e^{-\delta(\lambda,p)/\mu}\left(-\frac{d\delta}{\mu dp}\right)$$

$\tau_d(\lambda)$

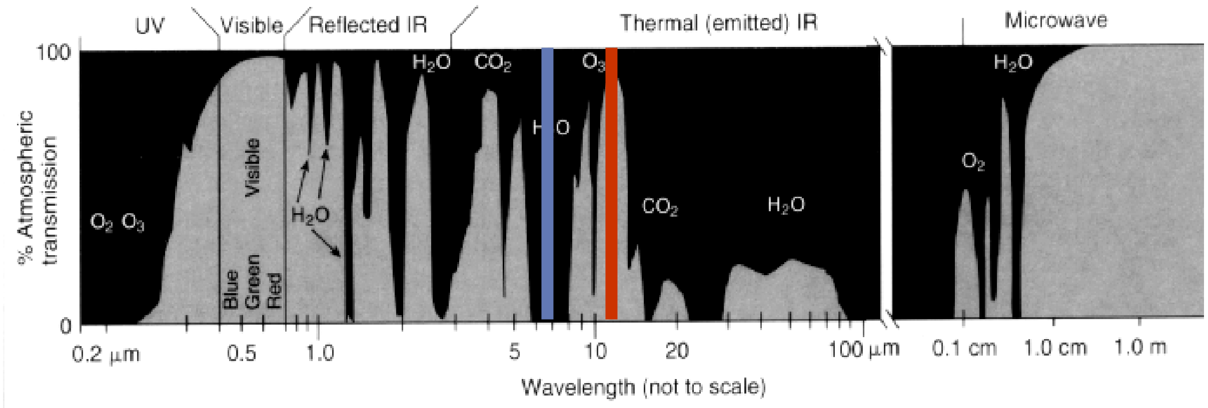
$\div$ -dp

$$L_t(\lambda,\theta,\varphi) = \varepsilon_s B(\lambda,T_s)e^{-\delta(\lambda)/\mu} + \int_0^{\delta(\lambda)} B(\lambda,T(z))\, e^{-\delta(\lambda,z)/\mu}\frac{d\delta}{\mu}$$

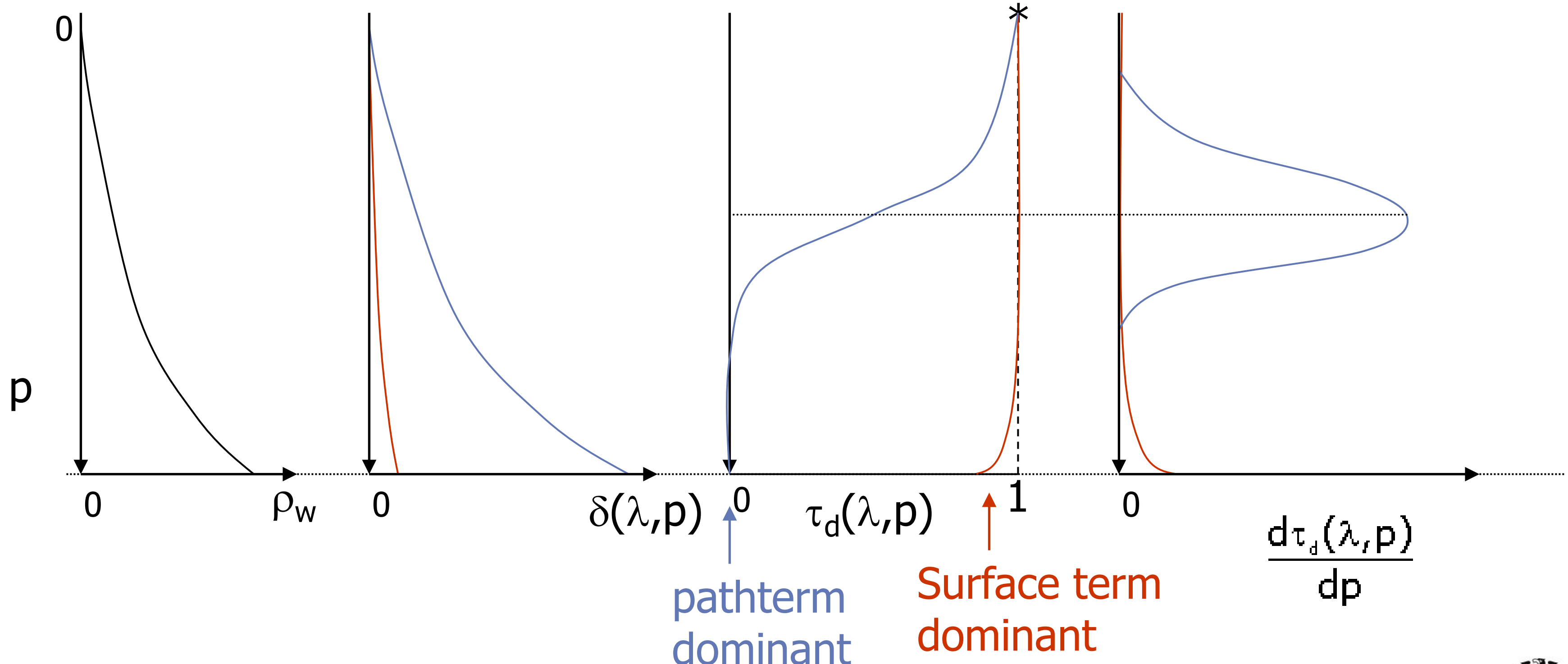
Thus, $$L_t(\lambda,\theta,\varphi) = \varepsilon_s(\lambda,\theta)B(\lambda,T_s)\tau_d(\lambda) + \int_{p_0}^{0} B(\lambda,T(p))\frac{d\tau_d(\lambda,p)}{dp}dp$$

$\frac{d\tau_d(\lambda,p)}{dp}$ = Weighting function → p where the atmospheric contribution to L_t is largest

Simplified Solution of RTE (Case #2)



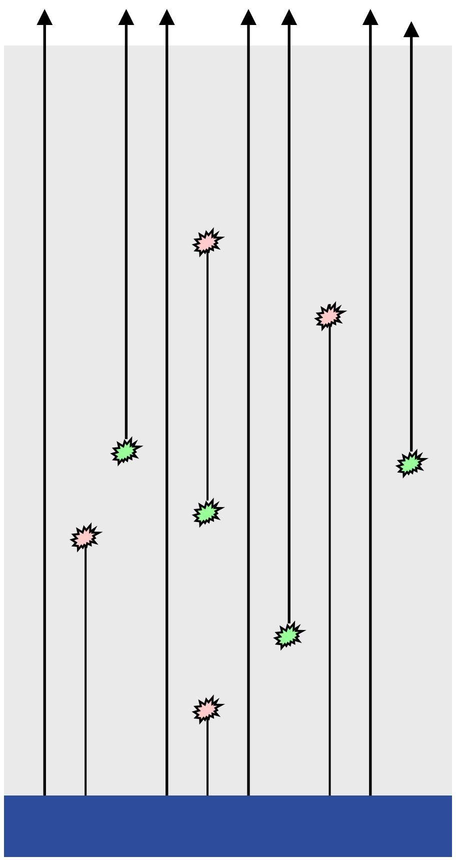
$$\tau_d = e^{-\delta(\lambda,p)/\mu}$$

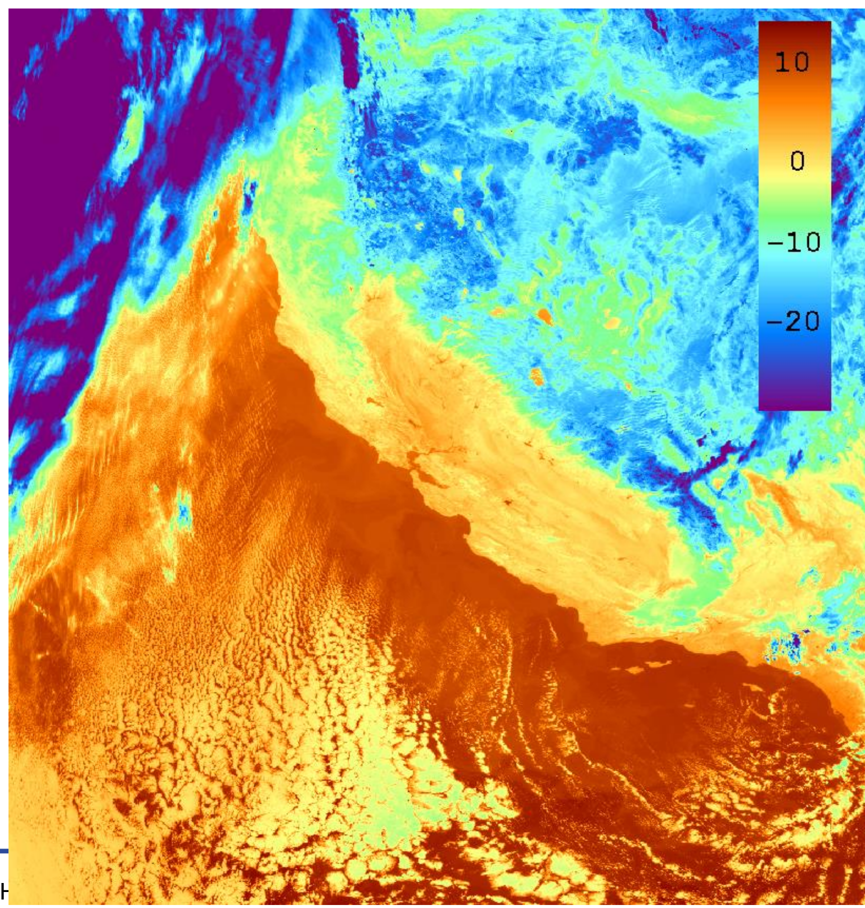


the Brightness temperature (TB) is defined as the Temperature which a black body has to have to be emit the same radiance (L_t) as measured by the satellite instrument,

Thus, if L_t is emitted from a blackbody, what would be his temperature?

L_t $\Rightarrow$ $$B(\lambda, T_B) = \frac{2hc^2}{\lambda^5\left(e^{hc/\lambda k T_B} - 1\right)}$$ and, $$T_B = \frac{hc^2}{\lambda k \ln\left(\frac{2hc^2}{\lambda^5 L_t}\right)}$$

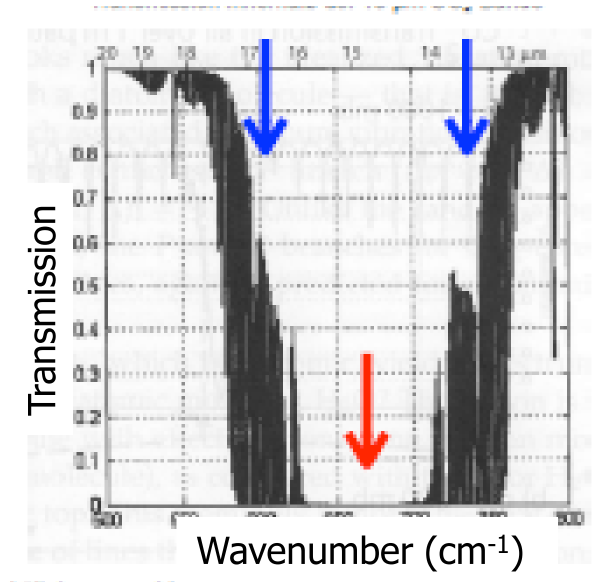




T_B
(AVHRR channel 4)

Vertical sonding (I)

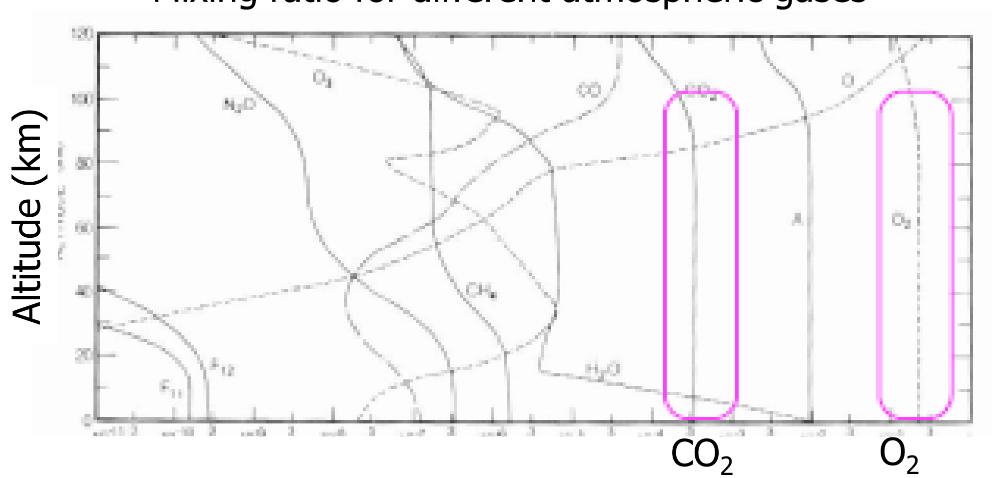
- The absorption within an absorption band decreases from the **centre** to the **edge**. The closer the wavelength is to the centre, the higher the radiation is absorbed in the atmosphere.
- In the TIR, the radiation arriving at the satellite contains information about the temperature and the gas profile of the respective absorbers



- The mixing ratio of CO2 and O2 is constant up to an altitude of around 100 km.

- The absorption properties of both gases are very well known.

Mixing ratio for different atmospheric gases



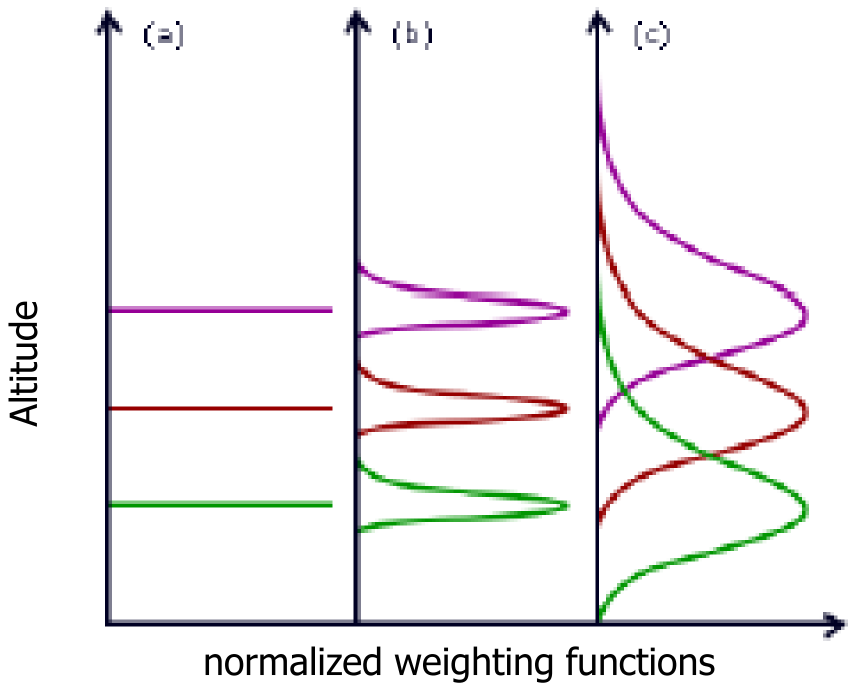
Vertical sounding (II)

Case (a)
the measured brightness temperature at one wavelength comes from one height. In this ideal case, the temperature at this height is obtained

Case (b)
the observed brightness temperature corresponds to the average temperature of a layer.
The layers are independent of each other.

Case (c)
the weighting functions overlap and the problem is underdetermined, i.e. in the best case the most probable temperature curve is obtained

Idealized weighting functions;
different colours correspond to different wavelengths



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A Simplified solution for Microwave will come afterwards.

Simplified Solution of RTE (Case #3)

Case #3

Source of photons only single scattering events
(in general always multi-scattering events)

$J = J_{scat}$

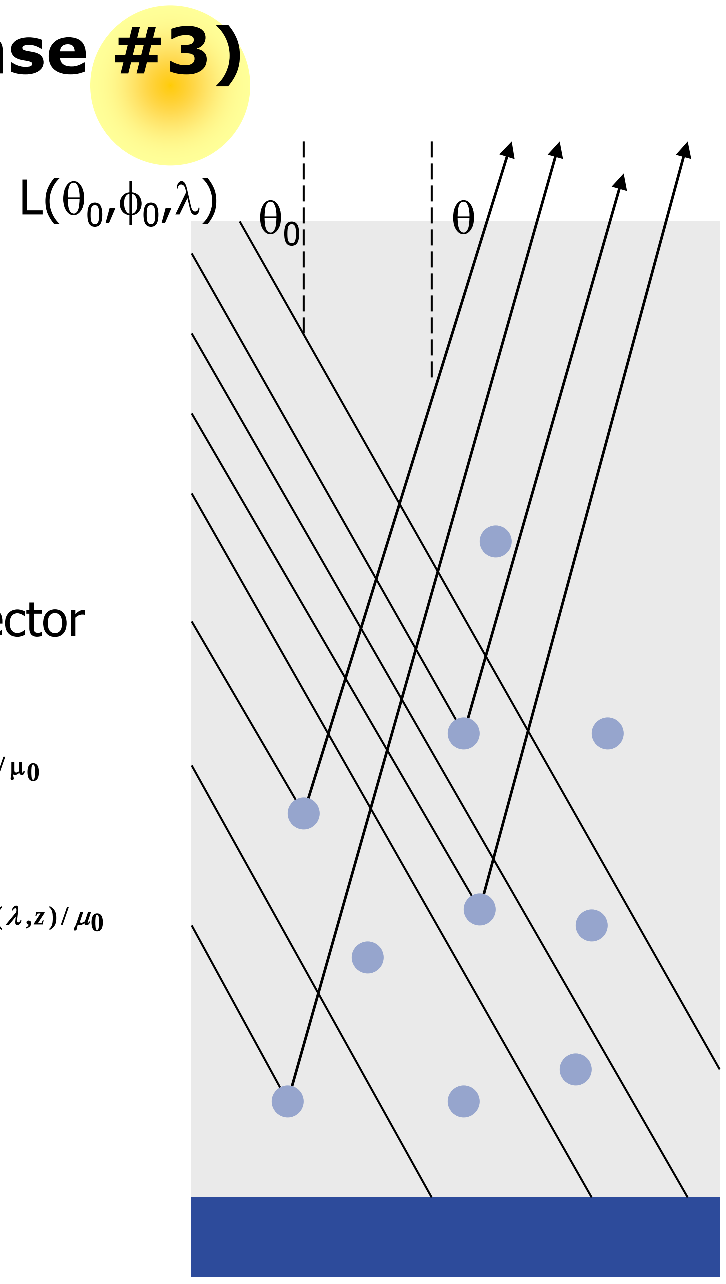
Single scattering means that every photon is only scattered once before it comes to the satellite detector
The only source of photons is the sun:

$$L(r',\lambda,X) = L(\theta_0,\phi_0,\lambda,X) = L(\theta_0,\phi_0,\lambda)e^{-\delta(\lambda,z)/\mu_0}$$

$$J_{scat} \cong \frac{\sigma_s(\lambda,z)}{4\pi} P(\theta_0,\theta,\varphi_0,\varphi;\lambda,X) L(\theta_0,\varphi_0,\lambda)e^{-\delta(\lambda,z)/\mu_0}$$

In which situation can this happen?

Optical thickness small <0.1 –
(e.g. optical thin cirrus, aerosol atmospheres)



Simplified Solution of RTE (Case #3)

At ToA:

$$L_t(\lambda,\theta,\varphi) = L_0(\lambda,\theta,\varphi)e^{-\delta(\lambda)/\mu} + \int_0^{\delta(z)} \frac{J(\lambda,z;\theta,\varphi)}{\sigma_e(\lambda,z)} e^{-\delta(\lambda,z)/\mu} \frac{d\delta}{\mu}$$

insert approximated scattering function:

$$L_t(\lambda,\theta,\varphi) = L_0(\lambda,\theta,\varphi)e^{-\delta(\lambda)/\mu} + \int_0^{\delta(z)} \frac{\sigma_s(\lambda,z)P(\Psi_s,\lambda,z)}{4\pi\sigma_e(\lambda,z)} L(\theta_0,\varphi_0,\lambda)e^{-\delta(\lambda,z)/\mu_0} e^{-\delta(\lambda,z)/\mu} \frac{d\delta}{\mu}$$

For a homogeneous layer:

$$L_t(\lambda,\theta,\varphi) = L_0(\lambda,\theta,\varphi)e^{-\delta(\lambda)/\mu} + \frac{\omega_0(\lambda)}{4\pi} P(\Psi_s,\lambda)L(\theta_0,\varphi_0,\lambda) \int_0^{\delta(\lambda)} e^{-\delta(\lambda,z)(1/\mu+1/\mu_0)} \frac{d\delta}{\mu}$$

$$L_t(\lambda,\theta,\varphi) = L_0(\lambda,\theta,\varphi)e^{-\delta(\lambda)/\mu} + \frac{\omega_0(\lambda)}{4\pi} P(\Psi_s,\lambda)L(\theta_0,\varphi_0,\lambda) \frac{(1-e^{-\delta(\lambda,z)(1/\mu+1/\mu_0)})}{(1/\mu+1/\mu_0)}$$

Radiance at the upper limit of the scattering layer consists of...

Radiance from the surface

The probability to be transmitted without a scattering event

Radiance from above

The probability of an interaction with a scattering particle (1- probability of no interaction)

The probability of an scattering event

The probability that the scattered radiance is scattered in direction of the satellite.

General Solution of RTE

Other Approximations, e.g.

$$L_t(\lambda,\theta,\varphi) = L_0(\lambda,\theta,\varphi)e^{-\delta(\lambda)/\mu} + \int_0^{\delta(z)} \frac{J(\lambda,z;\theta,\varphi)}{\sigma_e(\lambda,z)} e^{-\delta(\lambda,z)/\mu} \frac{d\delta}{\mu}$$

Substitute in scattering funcion

$$L_t(\lambda,\theta,\varphi) = L_0(\lambda,\theta,\varphi)e^{-\delta(\lambda)/\mu} + \int_0^{\delta(z)} \frac{\sigma_s \frac{1}{4\pi} \int_{4\pi} P(r',r,\lambda,X)L(r',\lambda,X)d\Omega'}{\sigma_e(\lambda,z)} e^{-\delta(\lambda,z)/\mu} \frac{d\delta}{\mu}$$

This equation implies that we know the radiance from all directions before we can calculate the radiance in direction r(θ,φ).

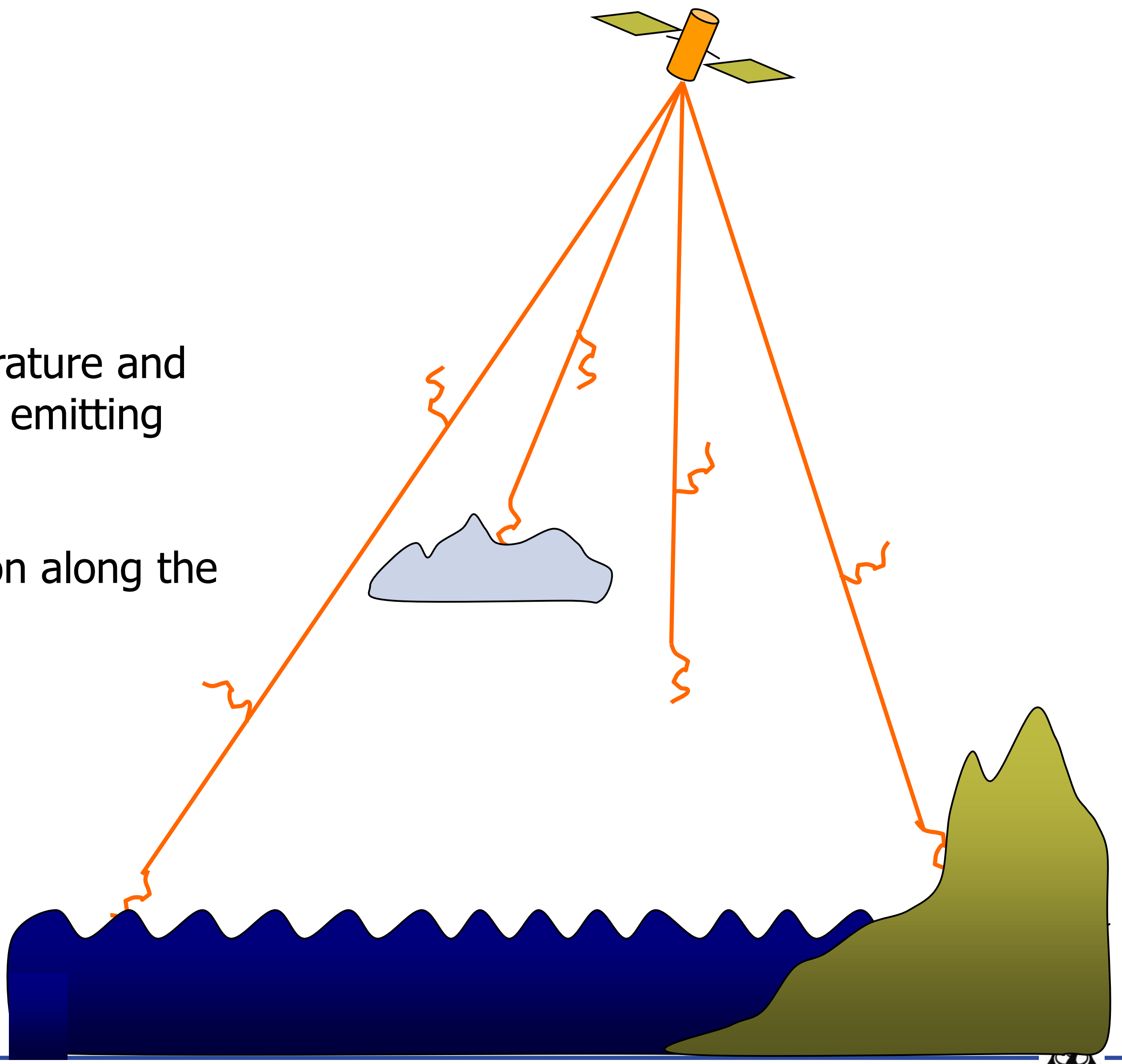
Schemes, how to solve these integral equations are part of the lecture on radiation.

Microwave radiative transport

Emitted Radiance

Only defined by temperature and emissivity properties of emitting source.

Weakened by absorption along the path to the sensor.



Microwave radiative transport (II)

In the microwave part of the electromagnetic spectrum…

… usually frequency (ν) (not any more wavelength) is used as spectral variable.
… frequencies small enough that Planckfunction is approximated by a linear function

$$B(\lambda, T) = \frac{2hc^2}{\lambda^5\left(e^{hc/\lambda kT} - 1\right)}$$

if $hc/\lambda kT << 1$, then $e^x \sim 1+x$ (for small x)

correct for $\lambda > 0.5$ cm or $\nu < 60$ GHz, if $T \sim 300$ K

$$B(\lambda,T) \sim (2ck/\lambda^4)\, T$$

$$B(\nu,T) = c/\nu^2\, B(\lambda,T) = (2k\nu^2/c^2)\, T$$

What are the consequences for radiative transport in the microwave?

Simple Solution: $L = \varepsilon_s\, B(\nu,T) = (2k\nu^2/c^2)\, \varepsilon_s\, T$

We define now the brightness temperature, $T_B = L\, c^2/2k\nu^2$

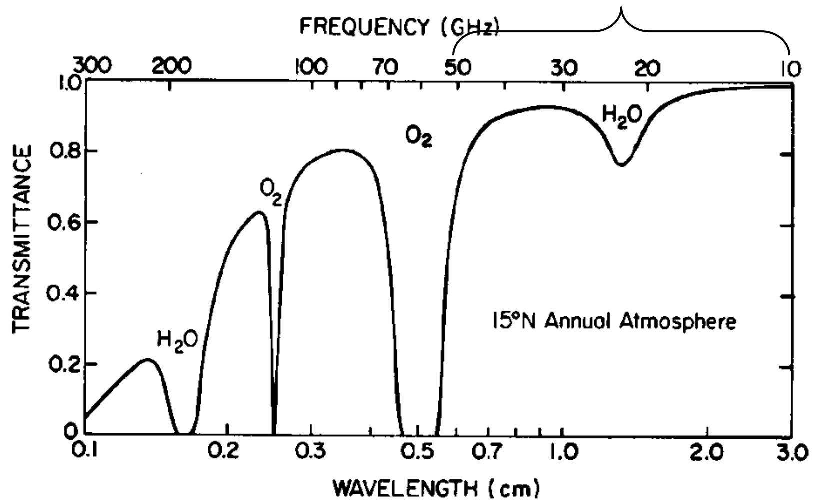
Then is $T_B = \varepsilon_s\, T$ (surface temperature, if blackbody)

… thus, in the RTE, T_B can be used as Intensity, and $B(\nu,T)$ can be substituted by T.

Microwave radiative transport, Case #4

Case #4

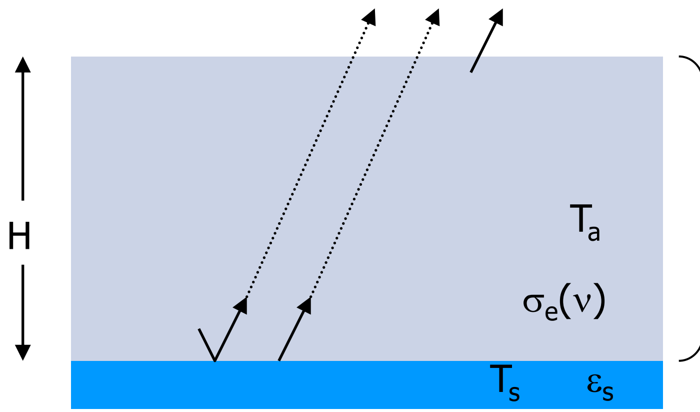
ν < 50GHz (relative absorptions window)



Optical thin homogeneous atmosphere

$$\sigma_e(\nu,z) = \sigma_e(\nu) = \sigma_a(\nu) + \sigma_s(\nu)$$

$$T(z) = T_{air}$$



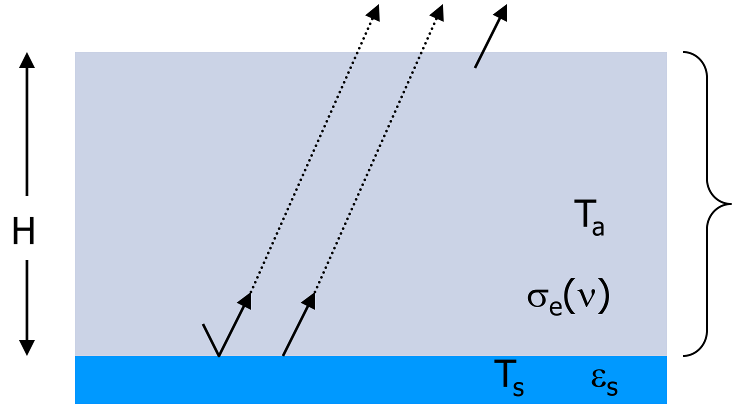
$\delta(\nu) = H\ \sigma_e(\nu)$

Surface radiation = Emission + Reflection

Sources along the path to the sensor only from Emission

Microwave, Case #4

$$L_t(\nu,\theta,\varphi) = L_0(\nu,\theta,\varphi)\tau_d(\nu) + \int_{\tau_d}^{1} B(\nu, T(p))\, d\tau_d$$



Upward emitted radiance from atmosphere:

$$\int_{\tau_d}^{1} B(\nu, T(p))\, d\tau_d = B(\nu, T_a)[1 - \tau_d(\nu)]$$

(This is also the downward emitted radiance of the atmosphere)

$1-\tau_d(\nu) = \alpha_a(\nu)$ (no atmospheric reflection)
$= \varepsilon_a(\nu)$

Total radiance at the top of the atmosphere is then:

$$L_t(\nu,\theta,\varphi) = \varepsilon_s(\nu,\theta,T_s,S)B(\nu,T_s)\tau_d(\nu) +$$

Emitted radiance from the surface transmitted to the top of the atmosphere

$$[1-\varepsilon_s(\nu,\theta,T_s,S)]B(\nu,T_a)[1-\tau_d(\nu)]\tau_d(\nu) +$$

Downward emitted radiance of the atmosphere, reflected at the surface and transmitted to the top of the atmosphere

$$B(\nu,T_a)[1-\tau_d(\nu)]$$

Upward emitted radiance of the atmosphere

Attention:
Surface emissivity depends on temperature, salinity, roughness of surface…